

Searching Saturation in eA Processes

V. P. Gonçalves ^{**}

*Instituto de Física e Matemática, Univ. Federal de Pelotas
Caixa Postal 354, 96010-900, Pelotas, RS, BRAZIL*

Abstract:

The high density effects should be manifest at small x and/or large nuclei. In this letter we consider the behavior of nuclear structure function F_2^A slope in the kinematic region which could be explored in the future eA colliders as a search of these effects. We verify that the high density implies that the maximum value of the slope occurs at large values of the photon virtuality, *i. e.* in a perturbative regime, and is dependent of the number of nucleons A and energy. Our conclusion is that the measurement of this observable will allow to explicit the saturation.

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^{0**}E-mail:barros@ufpel.tche.br

The physics of high-density QCD (hdQCD) has become an increasingly active subject of research, both from experimental and theoretical points of view. Presently, and in the near future, the collider facilities such as the DESY collider HERA (ep , eA), Fermilab Tevatron ($p\bar{p}$, pA), BNL Relativistic Heavy Ion Collider (RHIC) (eA , AA), and CERN Large Hadron Collider (LHC) ($p\bar{p}$, AA) will be able to probe new regimes of dense quark gluon matter at very small Bjorken x or/and at large A , with rather different dynamical properties. In these experiments, be it because of high energies in ep collisions or because of intrinsically higher numbers of partons in eA and AA collisions, QCD effects are dominated by the large number of gluons involved. The description of these processes is directly associated with a correct gluonic dynamics in this kinematical region.

Theoretically, at small x and/or large A we expect the transition of the regime described by the linear dynamics (DGLAP, BFKL) (For a review, see e.g. Ref. [1]), where only the parton emissions are considered, for a new regime where the physical process of recombination of partons become important in the parton cascade and the evolution is given by a nonlinear evolution equation. This regime is characterized by the limitation on the maximum phase-space parton density that can be reached in the hadron wavefunction (parton saturation) and very high values of the QCD field strength $F_{\mu\nu} \approx 1/g$ [2]. In this case, the number of gluons per unit phase space volume practically saturates and at large densities grows only very slowly (logarithmically) as a function of the energy. At this moment, there are many approaches in the literature that propose distinct evolution equations for the description of the gluon distribution in high density limit [3, 4] [5, 6]. In general these evolution equations resum powers of the function $\kappa(x, Q^2) \equiv \frac{3\pi^2\alpha_s A}{2Q^2} \frac{xg(x, Q^2)}{\pi R_A^2}$, which represents the probability of gluon-gluon interaction inside the parton cascade, matching

- the DLA limit of the DGLAP evolution equation in the limit of low parton densities ($\kappa \rightarrow 0$);
- the GLR equation and the Glauber-Mueller formula as first terms of the high density effects.

The main differences between these approaches occurs in the limit of very large densities, where all powers of κ should be resummed. Although the

complete demonstration of the equivalence between these formulations in the region of large κ is still an open question, some steps in this direction were given recently [7, 8].

Considering that the condition $\kappa = 1$ specifies the critical line, which separates between the linear regime $\kappa \ll 1$ and the high density regime $\kappa \gg 1$, we can define the saturation momentum scale Q_s given by

$$\kappa = 1 \Rightarrow Q_s^2(x; A) = \frac{3\pi^2\alpha_s A}{2} \frac{xg(x, Q_s^2(x; A))}{\pi R_A^2} , \quad (1)$$

below which the gluon densities reach their maximum value (saturates). At any value of x there is a value of $Q^2 = Q_s^2(x)$ in which the gluonic density reaches a sufficiently high value that the number of partons stops the growth. This scale depends on the energy of the process and the atomic number of the colliding nuclei, determining the typical intrinsic momenta associated with quanta in the nuclear wavefunction. These quanta go on shell in a collision, and eventually produce a large multiplicity of particles. At very high energies, the saturation scale is the only scale in the problem, and one can estimate that the typical momenta of the particles produced in the collision is at this scale. If this momenta is large enough, one can approach the saturation regime using perturbation theory. Recently, the high density effects in AA collisions were considered [9], verifying that the equilibration time, the initial temperature and the chemical potential have a strong functional dependence on the initial gluon saturation scale Q_s .

The deeply inelastic scattering experiments at HERA revealed that structure functions grows rapidly at small x and large Q^2 . Therefore, at some value of x , for a fixed Q^2 , it is expected that the parton distribution will saturate, leading to a weaker growth of the structure function. However, although these high density effects should be present in the ep HERA kinematical region, the current limited x range available at HERA makes it difficult to distinguish between the predictions of the linear and nonlinear dynamics. Basically, the same data compatible with the high density approaches can be described from a different point of view without the nonlinear effects in the standard DGLAP evolution equation for $Q^2 > 1 \text{ GeV}^2$ and the soft phenomenology for $Q^2 < 1 \text{ GeV}^2$ [10]. In Ref. [11] the authors has analyzed the position of the critical line, verifying that going along the critical line from $x = 10^{-4}$ to $x = 10^{-5}$ the saturation scale increases from approximately 1 GeV^2 up to 2 GeV^2 , *i. e.* the saturation scale is approximately constant in

the ep HERA kinematical region, allowing that the high density effects can be absorbed to a large extent in the initial conditions of the linear dynamics [12, 13].

Here we analyze the high density effects in eA processes, where we can study the dynamics of QCD at high densities and at zero temperature, raising questions complementary to those addressed in the search for a quark-gluon plasma in high-energy heavy ion collisions. The nucleus in this process serves as an amplifier for nonlinear phenomena expected in QCD at small x , obtaining at the assessable energies at HERA and RHIC with an eA collider the parton densities which would be probed only at energies comparable to LHC energies with an ep collider. Our goal is to address the boundary region between the linear and nonlinear dynamics and identify possible signatures in the behavior of the nuclear structure function slope. As the behavior of this quantity is strongly dependent of the gluon distribution, we expect distinct behaviors at $Q^2 > Q_s^2$ and $Q^2 < Q_s^2$ with the transition point dependent of the values of x and A . We assume that in the boundary region, the parton density is sufficiently large to invalid the descriptions which use the linear dynamics, but small to consider a general approach which resum all powers of κ . Basically, we will consider the high density effects in the kinematical region where the predictions using the Glauber-Mueller approach [4] are a good approximation, and our analysis in principle is not model dependent. This study is motivated by our results in Ref. [14], where we show that the deep inelastic scattering on nuclear targets is a very good place to look for saturation if we consider the behavior of the nuclear structure function slope, and demonstrate the distinct predictions from DGLAP and high density approaches. Here we extend the previous analyses for the energies of eA processes at RHIC and at HERA, as well as for the possible colliding nuclei in these experiments, analyzing the x and A dependence of the saturation scale (For a related discussion see Ref. [15]).

The deep inelastic scattering $eA \rightarrow e + X$ is characterized by a large electron energy loss ν (in the target rest frame) and an invariant momentum transfer $q^2 \equiv -Q^2$ between the incoming and outgoing electron such that $x = Q^2/2m_N\nu$ is fixed. It is usually interpreted in a frame where the nucleus is going very fast. In this case the high density effects are the result of an overlap in the longitudinal direction of the parton clouds originated from different bound nucleons [3]. It corresponds to the fact that small x partons cannot be localized longitudinally to better than the size of the nucleus.

Thus low x partons from different nucleons overlap spatially creating much larger parton densities than in the free nucleon case. This leads to a large amplification of the nonlinear effects expected in QCD at small x . In the target rest frame, the electron-nucleus scattering can be visualized in terms of the propagation of a small $q\bar{q}$ pair in high density gluon fields through much larger distances than it is possible with free nucleons. In terms of Fock states we then view the eA scattering as follows: the electron emits a photon ($|e\rangle \rightarrow |e\gamma\rangle$) with $E_\gamma = \nu$ and $p_{t\gamma}^2 \approx Q^2$, after the photon splits into a $q\bar{q}$ ($|e\gamma\rangle \rightarrow |eq\bar{q}\rangle$) and typically travels a distance $l_c \approx 1/m_N x$, referred as the 'coherence length', before interacting in the nucleus. For small x , the photon converts to a quark pair at a large distance before it interacts with the target. Consequently, the space-time picture of the DIS in the target rest frame can be viewed as the decay of the virtual photon at high energy (small x) into a quark-antiquark pair long before the interaction with the target. The $q\bar{q}$ pair subsequently interacts with the target. It allows to factorize the total cross section between the wavefunction of the photon and the interaction cross section of the quark-antiquark pair with the target. The photon wavefunction is calculable and the interaction cross section is modelled. Moreover, the cross sections for transverse and longitudinal photons are most conveniently written in a mixed representation. The two transverse directions are treated in the coordinate space, while the longitudinal direction is described in the momentum representation. Let \vec{r}_t be the two dimensional vector pointing from the quark to the antiquark in the transverse plane and z the fraction of photon energy ν carried by the quark. The momentum fraction of the antiquark is then $1 - z$. The nuclear structure function reads [16]

$$F_2^A(x, Q^2) = \frac{Q^2}{4\pi\alpha_{em}} \int dz \int \frac{d^2\vec{r}_t}{\pi} |\Psi(z, \vec{r}_t)|^2 \sigma^{q\bar{q}+A}(z, \vec{r}_t) , \quad (2)$$

where $\Psi(z, \vec{r}_t)$ is the light-cone wavefunction for the transition $\gamma^* \rightarrow q\bar{q}$ and we have assumed the dominance of the transverse photon polarization. The cross section for scattering a $q\bar{q}$ - dipole off the nucleus is denoted by $\sigma^{q\bar{q}+A}(z, \vec{r}_t)$. In the pure perturbative regime the reaction is mediated by single gluon exchange which changes into multi-gluon exchange when the saturation region is approached.

We estimate the high density effects considering the Glauber multiple scattering theory [17], which was probed in QCD [18]. The nuclear collision is analyzed as a succession of independent collisions of the probe with individual

nucleons within the nucleus, which implies that

$$F_2^A(x, Q^2) = \frac{Q^2}{4\pi\alpha_{em}} \int dz \int \frac{d^2\vec{r}_t}{\pi} |\Psi(z, \vec{r}_t)|^2 \int \frac{d^2\vec{b}_t}{\pi} 2 [1 - e^{-\sigma^{\bar{q}q+N}(z, \vec{r}_t)S(\vec{b}_t)}] , \quad (3)$$

where \vec{b}_t is the impact parameter, $S(\vec{b}_t)$ is the profile function and $\sigma^{\bar{q}q+N}$ is the dipole cross section off the nucleons inside the nucleus, which is proportional to the pair separation squared r_t^2 and the nucleon gluon distribution $xg(x, 1/r_t^2)$. The expression (3) represents is the Glauber-Mueller formula for the nuclear structure function (see [4] for details). The main characteristic of the Glauber-Mueller formula is that for a large Q^2 it reduces to the standard small x DGLAP expression, while at small Q^2 it goes to zero as $Q^2 \log Q^2$, predicting a transition in the behavior of F_2^A at an intermediate value of Q^2 . A similar expression was used in Ref. [11] for a phenomenological analysis of the ep process, disregarding the geometrical structure of the collision, the \vec{b}_t dependence, and assuming $xg \propto x^{-\lambda}$ ($\lambda > 0$), resulting a very good description of the HERA data.

Using a gaussian profile function, we can derive the slope of the nuclear structure function directly from the expression (3), resulting [14]

$$\frac{dF_2^A(x, Q^2)}{d \log Q^2} = \frac{R_A^2 Q^2}{2\pi^2} \sum_1^{n_f} \epsilon_i^2 \{C + \ln(\kappa_q(x, Q^2)) + E_1(\kappa_q(x, Q^2))\} , \quad (4)$$

where $\kappa_q = (2\alpha_s A/3R_A^2) \pi r_t^2 xg(x, \frac{1}{r_t^2})$, A is the number of nucleons, R_A^2 is the mean nuclear radius, C is the Euler constant and E_1 is the exponential function (see [14] for details). The expression (4) predicts the x , Q^2 and A dependence of the high density effects for the F_2^A slope. Similarly to the F_2^A we expect that a turnover, in the behavior of the F_2^A slope, should occur for the saturation scale.

Here we estimate the high density effects for the F_2^A slope in the kinematic regions which could be explored in eA colliders at HERA and RHIC, as well as for some typical values of the number of nucleons A . We use as input in our calculations the GRV95 parameterization [19] for the nucleon gluon distribution, since we expect that the nonlinear corrections to the DGLAP evolution equation probably seen at HERA are not obscured by it (see Ref. [20] for a full analysis).

As discussed in Ref. [21], in a first moment the experiments should be carried with nuclear targets with the ratio Z/A equal to 1/2. Hence the

energy for each nucleon in a deep inelastic collision will be half that in an ep collision. For instance, for eA processes at HERA, we will have that the energy per nucleon will be about 410 GeV . Assuming the current value of 27.6 GeV for electron energy results $W \approx 110\text{ GeV}$ at HERA and $W \approx 75$ at RHIC, where energy/nucleon = 200 GeV . Therefore, we will consider the possibility of electron scattering with carbon ($A = 12$) and sulfur ($A = 32$) targets, which satisfy the above condition, and extrapolate our analysis for the possibility of collisions with Au targets ($A = 197$).

Following [11], we consider that the Bjorken variable x and the photon virtuality Q^2 are related by the expression $x = Q^2/W^2$, where W is the γ^*p c.m. energy, and calculate the x and Q^2 dependences for W and A fixed. This approach is inspired by the recent analysis carried by ZEUS [22] to stipulate the deviation from the conventional perturbative QCD framework at low values of Q^2 . The Figs. 1 and 2 shows the logarithmic Q^2 slope of F_2^A plotted for fixed W^2 and different A as a function of Q^2 and x , respectively. The remarkable property of the plots is the presence of a distinct maximum for each slope, dependent of the energy and the number of nucleons considered. We can see that at fixed A , if we increase the value of the energy, the maximum value of the slope occur at larger values of Q^2 and smaller values of x . A similar behavior for the proton structure function slope was obtained in Ref. [11]. The remarkable properties of the collisions with nuclei targets are the large values of Q^2 for the turnover of F_2^A slope and the large shift of the turnover at large values of Q^2 with the growth of the energy. In Table 1 we present explicitly the A and W dependences of the turnover in F_2^A slope. We verify that, at fixed W , the turnover is displaced at larger values of x and Q^2 if we increase the number of nucleons A . These behaviors can be understood intuitively. The turnover is associated with the regime in which the partons in the nucleus form a dense system with mutual interactions and recombinations, with a transition between the linear and nonlinear regime at the saturation scale [Eq. (1)]. As the partonic density growth at larger values of the number of nucleons A and smaller values of x we have that, at fixed A , the saturation scale Q_s^2 will increase at small values of x , since this is directly proportional to the gluon distribution. Moreover, at fixed energy W , the same density at $A = 12, 32, 197$ will be obtained at larger values of x and Q^2 . These properties of the high density effects are verified in the F_2^A slope. The main result of our analysis is that the saturation should occur already at rather small distances (large Q_s^2) well below where soft dynamics

is supposed to set in, justifying the use of perturbative QCD to approach a highly dense system.

Our main conclusion is that deeply inelastic scattering of electrons off nuclei at high energies can determine whether parton distributions saturate. The analysis of the nuclear structure function slope at eA HERA and/or RHIC energies will allow to establish the presence of a high density system and the behavior of the saturation scale. Moreover, we predict that the transition between the linear and nonlinear regimes in eA processes at high energies will occur in a perturbative regime, justifying the current pQCD approaches. We hope that our estimates will be useful for planning of future eA experiments.

Acknowledgments

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Tables

	W = 50 GeV		W = 75 GeV		W = 115 GeV	
	x	Q^2	x	Q^2	x	Q^2
$A = 12$	0.620×10^{-3}	1.55	0.382×10^{-3}	2.15	0.231×10^{-3}	3.05
$A = 32$	0.740×10^{-3}	1.85	0.462×10^{-3}	2.6	0.287×10^{-3}	3.8
$A = 197$	0.11×10^{-2}	2.75	0.649×10^{-3}	3.65	0.389×10^{-3}	5.15

Table 1: The A and W dependences of the turnover predicted in the F_2^A slope.

Figure Captions

Fig. 1: The logarithmic Q^2 slope of F_2^A as function of the variable Q^2 at different values of A and W . See text.

Fig. 2: The logarithmic Q^2 slope of F_2^A as function of the variable x at different values of A and W . See text.

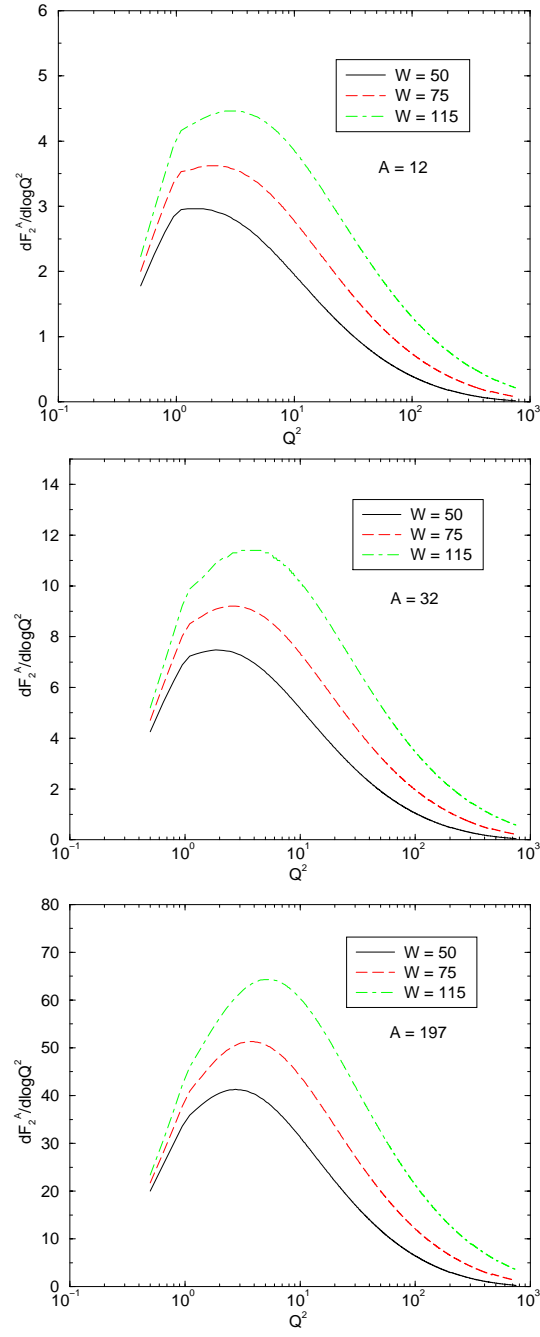


Figure 1:

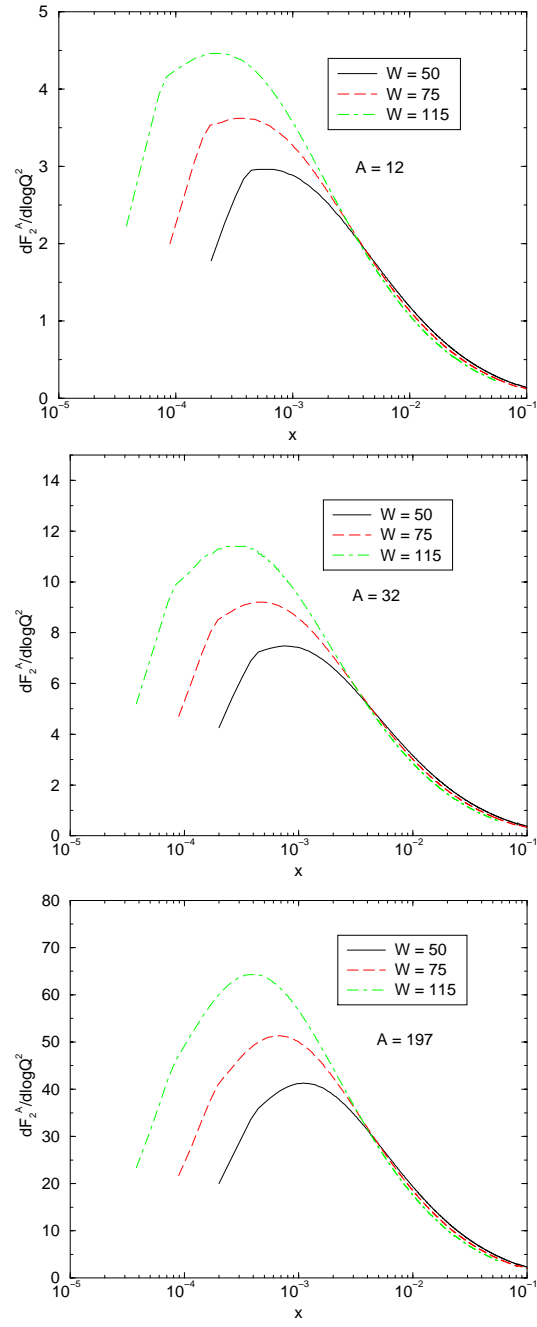


Figure 2: